Boundary Integral Equations and Block Jacobi Preconditioner

Waves 2019

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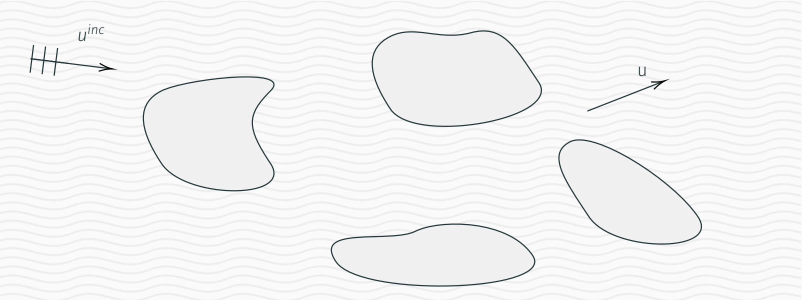
Time-Harmonic Acoustic Multiple Scattering



Boundary Integral Equations + Time Harmonic

- **Multiple scattering:** coupled problem
- **Large number of obstacles** + **High frequency** $(\lambda \ll L)$ = Direct solver (almost) *kaputt*
- **Indefiniteness:** Iterative solver slow or *kaputt*

Acceleration Techniques

- **Compression** (FMM or H-Matrix): iterative solver
- **Few Algebraic Preconditioners**: SPAI, ...
- **Analytic Preconditioners**: GCFIE, ...

Multiple Scattering Context: Single-Scattering or *Block-Jacobi*

$$\begin{pmatrix} S_{1,1} & S_{1,2} \\ S_{2,1} & S_{2,2} \end{pmatrix}$$

Acceleration Techniques

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- **Few Algebraic Preconditioners**: SPAI, ...
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Multiple Scattering Context: Single-Scattering or *Block-Jacobi*

$$\begin{pmatrix} S_{1,1}^{-1} & 0 \\ 0 & S_{2,2}^{-1} \end{pmatrix} \begin{pmatrix} S_{1,1} & S_{1,2} \\ S_{2,1} & S_{2,2} \end{pmatrix} = \underbrace{\begin{pmatrix} Id & S_{1,1}^{-1} S_{1,2} \\ S_{2,2}^{-1} S_{2,1} & Id \end{pmatrix}}_{\mathbb{I} + \mathbb{M}}$$

Related Works

Reflection Method (Block-Jacobi iterative method)

$$X^{n+1} = \mathbb{M}X^n + b$$

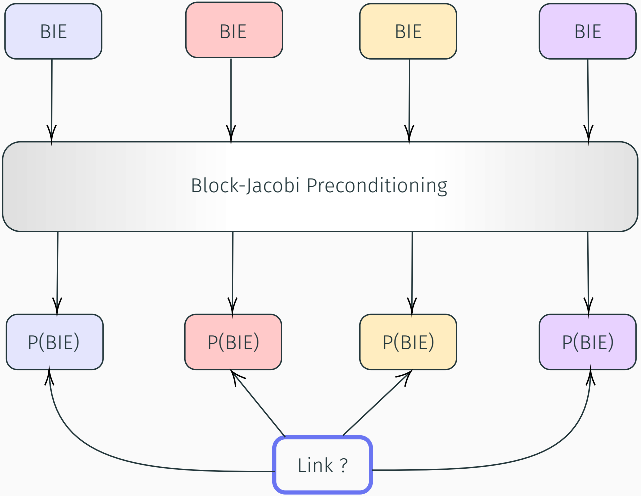
- **Laurent et. al. (2017)** : mathematical framework unification
- **Balabane**: proof and conditions of convergence for Helmholtz (*boundary decomposition method*)

Lax-Foldy Model (approximate the successive reflections)

$$(\mathbb{I} + \mathbb{M})^{-1} \simeq \sum_{\ell=0}^{L} (-\mathbb{M})^{\ell}$$

- **Cassier and Hazard (2013)**: circular obstacles
- **Challa and Sini (2018)**: arbitrary shape obstacles

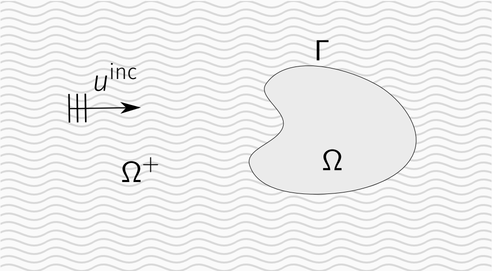
This talk: *Horizontal* study



The link (spoiler alert)

PBIEs are **equal** or **similar** (equal up to an invertible operator)

Direct BIE (*Null Field Method*)



PDE (Helmholtz)

$$\begin{cases} (\Delta + k^2)u = 0 & (\Omega^+) \\ u = 0 & (\Gamma) \\ (u - u^{\mathrm{inc}}) \text{ radiating} \end{cases}$$

Integral Representation

$$u = u^{\mathrm{inc}} - \mathscr{S}\,(\partial_{\mathsf{n}} u)|_\Gamma - \underbrace{\mathscr{D}\,u|_\Gamma}_{=0}$$

Direct BIE (*Null Field Method*)[1]

- u sough as a single-layer potential of **density** σ

$$u = u^{\text{inc}} + \mathscr{S}\sigma = u^{\text{inc}} + \int_\Gamma G(\cdot, y)\sigma(y)\mathrm{d}y \qquad (\Omega^+)$$

- Fictious wave u^- inside Ω:

$$\begin{cases} u & = & u^{\text{inc}} + \mathscr{S}\sigma & (\Omega^+) \\ u^- & = & u^{\text{inc}} + \mathscr{S}\sigma & (\Omega) \end{cases}$$

- Jump relations on Γ: $\sigma = \partial_n u^-|_\Gamma - \partial_n u|_\Gamma$
- If $u^- \equiv 0$ in Ω then $\sigma = -\partial_n u|_\Gamma$ (= *Cauchy Data*)

[1] See *e.g* **A. Bendali and M. Fares**, *Boundary Integral Equations Methods in Acoustics*, 2008

How to make $u^- \equiv 0$?

$$u^- = u^{\text{inc}} + \mathscr{S}\sigma, \quad (\Omega) \iff \begin{cases} (\Delta + k^2)u^- = 0 & (\Omega) \\ Au^- = 0 & (\Gamma) \end{cases}$$

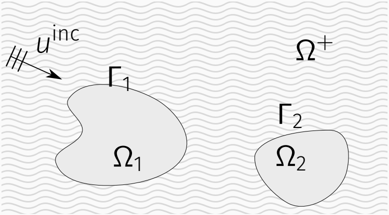
A is an Interior Trace Operator that we chose

Value of A	Resulting BIE	Name
Trace $\cdot\|_\Gamma$	$S\sigma = -u^{\text{inc}}\|_\Gamma$	(EFIE)
Normal Trace $\partial_n \cdot\|_\Gamma$	$(I/2 - K)\sigma = -\partial_n u^{\text{inc}}\|_\Gamma$	(MFIE)
$\alpha \cdot\|_\Gamma + (1-\alpha)\, \partial_n \cdot\|_\Gamma$	αEFIE $+ (1-\alpha)$MFIE	(CFIE)

with: $S = \mathscr{S}|_\Gamma$ $\quad$ $I/2 - K = \partial_n \mathscr{S}|_\Gamma$

Generic BIE: $(A\mathscr{S}\sigma =)\ S^A\sigma = b^A$ (with $\sigma = -\partial_n u|_\Gamma$)

Multiple Scattering: what change?



Split the potential

$$u = u^{\text{inc}} + \sum_q \int_{\Gamma_q} G(\cdot, y)\, \sigma|_{\Gamma_q}(y)\mathrm{d}y$$
$$= u^{\text{inc}} + \sum_q \mathscr{S}_q \sigma_q$$

Fictitious fields inside Ω_p: $\quad u_p^- = u^{\text{inc}} + \sum_q \mathscr{S}_q \sigma_q$

Apply trace operator A_p on every Γ_p:

$$\begin{cases} A_1 u^- = 0 \\ A_2 u^- = 0 \end{cases} \iff \begin{pmatrix} S_{1,1}^A & S_{1,2}^A \\ S_{1,2}^A & S_{2,2}^A \end{pmatrix} \begin{pmatrix} \sigma_1 \\ \sigma_2 \end{pmatrix} = \begin{pmatrix} b_1^A \\ b_2^A \end{pmatrix}$$

with $S_{p,q}^A = A_p \mathscr{S}_q$

Single Scattering (or *Block Jacobi*) Preconditioning

$$\underbrace{\begin{pmatrix} \left(S_{1,1}^A\right)^{-1} & 0 \\ 0 & \left(S_{2,2}^A\right)^{-1} \end{pmatrix}}_{\widehat{S}^A} \underbrace{\begin{pmatrix} S_{1,1}^A & S_{1,2}^A \\ S_{2,1}^A & S_{2,2}^A \end{pmatrix}}_{S^A} \begin{pmatrix} \sigma_1 \\ \sigma_2 \end{pmatrix} = \begin{pmatrix} \left(S_{1,1}^A\right)^{-1} b_1^A \\ \left(S_{2,2}^A\right)^{-1} b_2^A \end{pmatrix}$$

Single Scattering (or *Block Jacobi*) Preconditioning

$$\underbrace{\begin{pmatrix} I_{1,1} & \left(S^A_{1,1}\right)^{-1} S^A_{1,2} \\ \left(S^A_{2,2}\right)^{-1} S^A_{2,1} & I_{2,2} \end{pmatrix}}_{\widehat{S}^A S^A} \begin{pmatrix} \sigma_1 \\ \sigma_2 \end{pmatrix} = \begin{pmatrix} \left(S^A_{1,1}\right)^{-1} & b^A_1 \\ \left(S^A_{2,2}\right)^{-1} & b^A_2 \end{pmatrix}$$

Proposition [Thierry, 2013] + Erata

Consider *two "trace operators" A and B*

If A_p *and* B_p ***only depends on*** Ω_p

Then $\widehat{S}^A S^A = \widehat{S}^B S^B$

Single Scattering (or *Block Jacobi*) Preconditioning

$$\underbrace{\begin{pmatrix} I_{1,1} & \left(S^A_{1,1}\right)^{-1} S^A_{1,2} \\ \left(S^A_{2,2}\right)^{-1} S^A_{2,1} & I_{2,2} \end{pmatrix}}_{\widehat{S}^A S^A} \begin{pmatrix} \sigma_1 \\ \sigma_2 \end{pmatrix} = \begin{pmatrix} \left(S^A_{1,1}\right)^{-1} b^A_1 \\ \left(S^A_{2,2}\right)^{-1} b^A_2 \end{pmatrix}$$

Proposition [Thierry, 2013] + Erata

Consider *two "trace operators" A and B*

If A_p *and* B_p ***only depends on*** Ω_p

Then $\widehat{S}^A S^A = \widehat{S}^B S^B$

It is enough to show $\left(S^A_{p,p}\right)^{-1} S^A_{p,q} = \left(S^B_{p,p}\right)^{-1} S^B_{p,q}$

Proof of Proposition

Lemma

$\forall u^{\text{inc}}, \qquad \left(S^A_{p,p}\right)^{-1} A_p\, u^{\text{inc}} = \left(S^B_{p,p}\right)^{-1} B_p\, u^{\text{inc}}$

Proof: (Single-)Scattering problem in $\Omega_p^+ = \mathbb{R}^3 \setminus \overline{\Omega_p}$

$$\begin{cases} \text{Helmholtz} & (\Omega_p^+) \\ v = 0 & (\Gamma_p) \\ (v - u^{\text{inc}}) \text{ outgoing} & \end{cases} \iff \begin{cases} v = u^{\text{inc}} + \mathscr{S}_p \mu_p \\ + \begin{cases} S^A_{p,p}\mu_p = A_p u^{\text{inc}} \\ \text{and} \\ S^B_{p,p}\mu_p = B_p u^{\text{inc}} \end{cases} \end{cases}$$

Proof of Proposition

Lemma (Recall)

$$\forall u^{\mathrm{inc}}, \qquad \left(S^A_{p,p}\right)^{-1} A_p \, u^{\mathrm{inc}} = \left(S^B_{p,p}\right)^{-1} B_p \, u^{\mathrm{inc}}$$

Proof: (Single-)Scattering problem in $\Omega_p^+ = \mathbb{R}^3 \setminus \overline{\Omega_p}$

$$\left\{ \begin{array}{lr} \text{Helmholtz} & (\Omega_p^+) \\ v = 0 & (\Gamma_p) \\ (v - u^{\mathrm{inc}}) \text{ outgoing} & \end{array} \right. \iff \left\{ \begin{array}{l} v = u^{\mathrm{inc}} + \mathscr{S}_p \mu_p \\ + \left\{ \begin{array}{l} S^A_{p,p}\mu_p = A_p u^{\mathrm{inc}} \\ \text{and} \\ S^B_{p,p}\mu_p = B_p u^{\mathrm{inc}} \end{array} \right. \end{array} \right.$$

Proof of Prop.: $\widehat{S}^A S^A = \widehat{S}^B S^B$

$$\begin{aligned} \forall p \neq q, \quad \left(S^A_{p,p}\right)^{-1} S^A_{p,q} &= \left(S^A_{p,p}\right)^{-1} A_p \, \mathscr{S}_q \\ &= \left(S^B_{p,p}\right)^{-1} B_p \, \mathscr{S}_q \\ &= \left(S^B_{p,p}\right)^{-1} S^B_{p,q} \end{aligned}$$

What about indirect BIE?

Brackage-Werner BIE (BW) vs. Direct BIE (EFIE)

BW and EFIE: **both obtained** by applying the **exterior trace**

	EFIE	BW
$u = \dots$	$u^{\text{inc}} + \mathscr{S}\sigma$	$u^{\text{inc}} + \mathscr{S}^{\text{BW}}\psi$
$(BC)\ u\|_\Gamma = 0$	$S\sigma = -\left. u^{\text{inc}} \right\|_\Gamma$	$S^{\text{BW}}\psi = -\left. u^{\text{inc}} \right\|_\Gamma$

$$\mathscr{S}^{\text{BW}} = -\eta\mathscr{S} - \mathscr{D} \qquad S^{\text{BW}} = -\eta S - K + I/2$$

Link between the two solutions

$$S\sigma = S^{\text{BW}}\psi \implies \sigma = S^{-1}S^{\text{BW}}\psi$$

Precond(BW) vs. Precond(EFIE)

$$\underbrace{\begin{pmatrix} S_{1,1}^{\mathrm{BW}} & S_{1,2}^{\mathrm{BW}} \\ S_{2,1}^{\mathrm{BW}} & S_{2,2}^{\mathrm{BW}} \end{pmatrix}}_{S^{\mathrm{BW}}} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} = \begin{pmatrix} b_1^{\mathrm{BW}} \\ b_2^{\mathrm{BW}} \end{pmatrix}$$

$$\underbrace{\begin{pmatrix} S_{1,1} & S_{1,2} \\ S_{2,1} & S_{2,2} \end{pmatrix}}_{S} \begin{pmatrix} \sigma_1 \\ \sigma_2 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$$

Precond(BW) vs. Precond(EFIE)

$$\underbrace{\begin{pmatrix} \left(S^{\mathrm{BW}}_{1,1}\right)^{-1} & 0 \\ 0 & \left(S^{\mathrm{BW}}_{2,2}\right)^{-1} \end{pmatrix}}_{\widehat{S}^{\mathrm{BW}}} \underbrace{\begin{pmatrix} S^{\mathrm{BW}}_{1,1} & S^{\mathrm{BW}}_{1,2} \\ S^{\mathrm{BW}}_{2,1} & S^{\mathrm{BW}}_{2,2} \end{pmatrix}}_{S^{\mathrm{BW}}} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} = \begin{pmatrix} \left(S^{\mathrm{BW}}_{1,1}\right)^{-1} b^{\mathrm{BW}}_1 \\ \left(S^{\mathrm{BW}}_{2,2}\right)^{-1} b^{\mathrm{BW}}_2 \end{pmatrix}$$

$$\underbrace{\begin{pmatrix} \left(S_{1,1}\right)^{-1} & 0 \\ 0 & \left(S_{2,2}\right)^{-1} \end{pmatrix}}_{\widehat{S}} \underbrace{\begin{pmatrix} S_{1,1} & S_{1,2} \\ S_{2,1} & S_{2,2} \end{pmatrix}}_{S} \begin{pmatrix} \sigma_1 \\ \sigma_2 \end{pmatrix} = \begin{pmatrix} \left(S_{1,1}\right)^{-1} b_1 \\ \left(S_{2,2}\right)^{-1} b_2 \end{pmatrix}$$

Precond(BW) vs. Precond(EFIE)

$$\underbrace{\begin{pmatrix} I_{1,1} & \left(S_{1,1}^{\mathrm{BW}}\right)^{-1} S_{1,2}^{\mathrm{BW}} \\ \left(S_{2,2}^{\mathrm{BW}}\right)^{-1} S_{2,1}^{\mathrm{BW}} & I_{2,2} \end{pmatrix}}_{\widehat{S}^{\mathrm{BW}} S^{\mathrm{BW}}} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} = \begin{pmatrix} \left(S_{1,1}^{\mathrm{BW}}\right)^{-1} b_1^{\mathrm{BW}} \\ \left(S_{2,2}^{\mathrm{BW}}\right)^{-1} b_2^{\mathrm{BW}} \end{pmatrix}$$

$$\underbrace{\begin{pmatrix} I_{1,1} & \left(S_{1,1}\right)^{-1} S_{1,2} \\ \left(S_{2,2}\right)^{-1} S_{2,1} & I_{2,2} \end{pmatrix}}_{\widehat{S} S} \begin{pmatrix} \sigma_1 \\ \sigma_2 \end{pmatrix} = \begin{pmatrix} \left(S_{1,1}\right)^{-1} b_1 \\ \left(S_{2,2}\right)^{-1} b_2 \end{pmatrix}$$

Precond(BW) vs. Precond(EFIE)

$$\underbrace{\begin{pmatrix} I_{1,1} & \left(S^{BW}_{1,1}\right)^{-1} S^{BW}_{1,2} \\ \left(S^{BW}_{2,2}\right)^{-1} S^{BW}_{2,1} & I_{2,2} \end{pmatrix}}_{\widehat{S}^{BW} S^{BW}} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} = \begin{pmatrix} \left(S^{BW}_{1,1}\right)^{-1} b^{BW}_1 \\ \left(S^{BW}_{2,2}\right)^{-1} b^{BW}_2 \end{pmatrix}$$

$$\underbrace{\begin{pmatrix} I_{1,1} & \left(S_{1,1}\right)^{-1} S_{1,2} \\ \left(S_{2,2}\right)^{-1} S_{2,1} & I_{2,2} \end{pmatrix}}_{\widehat{S} S} \begin{pmatrix} \sigma_1 \\ \sigma_2 \end{pmatrix} = \begin{pmatrix} \left(S_{1,1}\right)^{-1} b_1 \\ \left(S_{2,2}\right)^{-1} b_2 \end{pmatrix}$$

Proposition [Thierry, 2013]

$$\left(S^{-1} S^{BW}\right)^{-1} \widehat{S} S \left(S^{-1} S^{BW}\right) = \widehat{S}^{BW} S^{BW}$$

Precond(BW) $\sim$ Precond(EFIE): Proof

Proposition (recall)

$$\left(S^{-1}S^{\mathrm{BW}}\right)^{-1}\widehat{S}S\left(S^{-1}S^{\mathrm{BW}}\right)=\widehat{S}^{\mathrm{BW}}S^{\mathrm{BW}}$$

Reverse engineering:

$$\left(S^{-1}S^{\mathrm{BW}}\right)^{-1}\widehat{S}S\left(S^{-1}S^{\mathrm{BW}}\right)=\widehat{S}^{\mathrm{BW}}S^{\mathrm{BW}} \qquad \cdot\times\left(S^{\mathrm{BW}}\right)^{-1}$$

$$\left(S^{-1}S^{\mathrm{BW}}\right)^{-1}\widehat{S}\,SS^{-1}=\widehat{S}^{\mathrm{BW}} \qquad SS^{-1}=I$$

$$\left(S^{-1}S^{\mathrm{BW}}\right)^{-1}\widehat{S}=\widehat{S}^{\mathrm{BW}} \qquad \text{rearrange}$$

$$\left(S^{\mathrm{BW}}\right)^{-1}S\widehat{S}=\widehat{S}^{\mathrm{BW}} \qquad S^{\mathrm{BW}}\times\cdot$$

$$S\widehat{S}\overset{?}{=}S^{\mathrm{BW}}\widehat{S}^{\mathrm{BW}}$$

Precond(BW) $\sim$ Precond(EFIE): $S\widehat{S} \overset{?}{=} S^{\mathrm{BW}}\widehat{S}^{\mathrm{BW}}$

	$(S\widehat{S})_{p,q}$	$(S^{\mathrm{BW}}\widehat{S}^{\mathrm{BW}})_{p,q}$
$p = q$	Id	Id
$p \neq q$	$\left(\mathscr{S}_q S_{q,q}^{-1}\right)\Big\vert_{\Gamma_p}$	$\left(\mathscr{S}_q^{\mathrm{BW}} S_{q,q}^{BW,-1}\right)\Big\vert_{\Gamma_p}$

For f_q living on Γ_q:

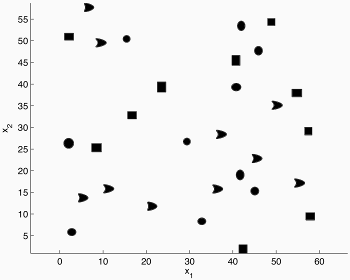
$$\begin{cases} w = \mathscr{S}_q S_{q,q}^{-1} f_q \\ w_{BW} = \mathscr{S}_q^{\mathrm{BW}} S_{q,q}^{BW,-1} f_q \end{cases}$$

are radiating solutions of the same single-scattering problem:

$$\begin{cases} (\Delta + k^2)v = 0 & (\Omega_q^+) \\ v = f_q & (\Gamma_q) \\ v \text{ is radiating} \end{cases}$$

Numerical illustration

Multiple Scattering Problem

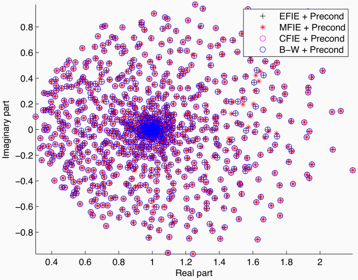


Context

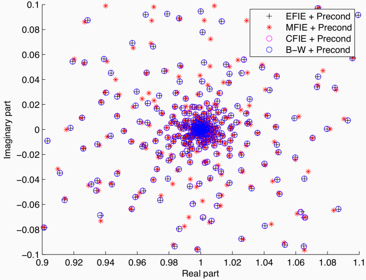
- 2d BEM
- 30 obstacles: size $\simeq 1$
- $\mathrm{dist} \geq 3$
- $k = 20$

What **should** we see?	What **would** be nice?
Same Matrices	Same convergence history
Same Eigenvalues	Faster convergence

Eigenvalues and Matrices



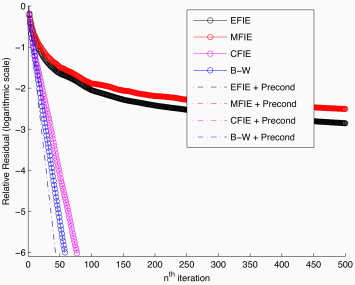
Eigenvalues



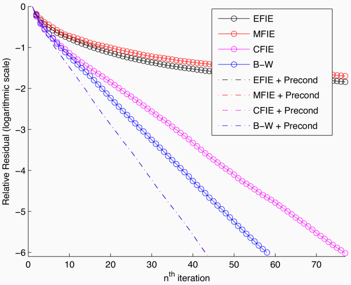
Zoom

- ✓ Matrices: Relative error ($\|\cdot\|_\infty$) = $\simeq 10^{-2}$
- ✓ Eigenvalues: Maximum relative error = 2.7%

GMRES: History of Convergence



History of convergence

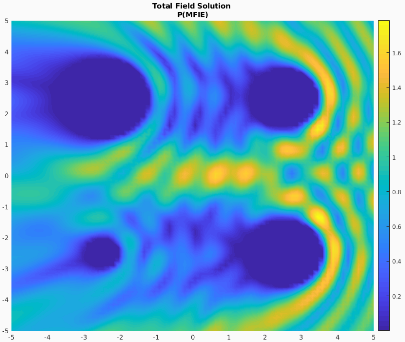


Zoom

- ✓ Superimposed to each others
- ✓ Faster convergence …

… But it's a test-case 🤷‍♂️

3D: 4 spheres (GypsiLab)



Context

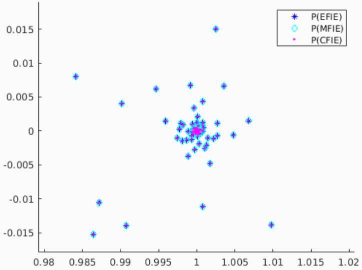
- 3d BEM
- 4 spheres: size $\simeq 1$
- $k = 4.4$

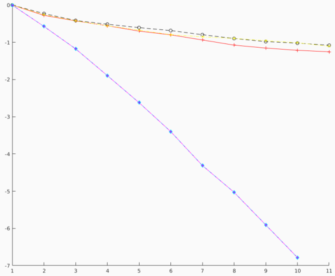
GypsiLab

- Open-source Matlab toolbox for FEM/BEM
- Quite easy to use (even without the documentation)

See the talk of F. Alouges on thursday

Eigenvalues and History of convergence



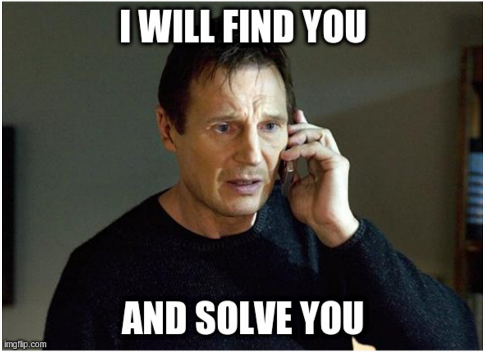


✓“Same” Eigenvalues

✓Same convergence rate

✓Same as 2D (no bad surprise)

What is the underlying BIE?



GCFIE - Single Scattering Case

Generalized CFIE[1]: $\mathrm{V}(\cdot|_\Gamma) + (\partial_\mathrm{n} \cdot|_\Gamma)$

Dirichlet-to-Neumann Map (DtN)

$$\mathrm{DtN}\,(f) = \partial_\mathrm{n}^+ w \text{ with } \begin{cases} \text{Helmholtz} & (\Omega^+) \\ w = f & (\Gamma) \\ w \text{ outgoing} & \end{cases}$$

"Trace" Operator: $\mathrm{E} = -\,\mathrm{DtN}\,(\cdot|_\Gamma) + (\partial_\mathrm{n} \cdot|_\Gamma)$

Nice Relation: $(\mathrm{E}\,\mathscr{S} =)\ S^\mathrm{E} = I$

"BIE":

$$I\sigma = -\,\mathrm{E}(u^{\mathrm{inc}})$$

[1] See *e.g.* X. Antoine and M. Darbas, *Generalized combined field integral equations for the iterative solution of the three-dimensional Helmholtz equation*, 2007

GCFIE - Multiple Scattering Case

Generalized CFIE: $\mathrm{V}(\cdot|_\Gamma) + (\partial_\mathrm{n} \cdot|_\Gamma)$

Dirichlet-to-Neumann Maps (DtN_p)

$$\mathrm{DtN}_p(f_p) = \partial_\mathrm{n}^+ w_p \text{ with } \begin{cases} \text{Helmholtz} & (\Omega_p^+) \\ w_p = f_p & (\Gamma_p) \\ w_p \text{ outgoing} & \end{cases}$$

"Trace" Operators: $\mathrm{E}_p = -\mathrm{DtN}_p(\cdot|_{\Gamma_p}) + (\partial_\mathrm{n} \cdot|_{\Gamma_p})$

Less but Still Nice Relation: ($\mathrm{E}_p \mathscr{S}_p =$) $S^\mathrm{E}_{p,p} = I_{p,p}$

Matrix:

$$\begin{pmatrix} I_{1,1} & S^\mathrm{E}_{1,2} \\ S^\mathrm{E}_{1,2} & I_{2,2} \end{pmatrix} \begin{pmatrix} \sigma_1 \\ \sigma_2 \end{pmatrix} = \begin{pmatrix} b^\mathrm{E}_1 \\ b^\mathrm{E}_2 \end{pmatrix}$$

This matrix sounds familiar!

Wait a minut...

Previous DtN can be used in multiple scattering case, so is the proposition wrong?

Back to the GCFIE

Global Dirichlet-to-Neumann Map (DtN)

$$\mathrm{DtN}(f) = \partial_{\mathrm{n}}^{+} w \text{ with } \begin{cases} \text{Helmholtz} & (\Omega^{+}) \\ w = f & (\Gamma) \\ w \text{ outgoing} & \end{cases}$$

(Single Scattering) “Trace” Operator: $\mathrm{E} = -\mathrm{DtN}(\cdot|_{\Gamma}) + (\partial_{\mathrm{n}} \cdot|_{\Gamma})$

(Multiple Scattering) “Trace” Operators: $\mathrm{E}_p\, g = (\mathrm{E}\, g)\Big|_{\Gamma_p}$

Matrix:

$$\begin{pmatrix} I_{1,1} & 0 \\ 0 & I_{2,2} \end{pmatrix} \begin{pmatrix} \sigma_1 \\ \sigma_2 \end{pmatrix} = \begin{pmatrix} b_1^{\mathrm{E}} \\ b_2^{\mathrm{E}} \end{pmatrix}$$

This matrix DOES NOT sounds familiar!

Reason: E_p depends on every other Ω_q

Conclusion

Conclusion

Block-Jacobi Preconditionner ...

- ✓ *Equalizes* [a lot of] direct BIEs
- ✓ *Similarizes* [a lot of] direct BIEs and BW

  $\longrightarrow$Extended to *every BIE* obtained *via* exterior Dirichlet
- ✓ Numerically illustrated (2D and 3D)
- ✓ Case of disks + Fourier: Mie Series = direct BIE

What next?

- ? Orthogonality needed
- ? Maxwell's / Elasticity: (probably) straightforward
- ? Spectrum of the BIE (GCFIE + DtN local):

$$\mathbb{I} + \mathbb{M} \text{ with } \mathbb{M} \text{ compact}$$

Thank you!